$$\frac{g_1^4}{4g_0^2}.$$

Since $g_1 \leqq g_0$, all points inside the circle can be shown to correspond to negative g_a, and this is physically inacceptable. Just as in the broad-band case, then[32] negative IF conductance is not obtained with a linear capacitor. Note, however, that one of the conditions, which is certainly not met in practice, is that the *LO* voltage waveform be symmetrical. To the author's knowledge, the more general case has not been investigated, although there is experimental evidence[33] that relaxation of this restriction will not change the picture.

[32] *Ibid*

[33] H. Q. North, *et al.*, "Welded Germanium Crystals," GE Rep., Contract OEMsr-262, Order No. DIC 178554; September 20, 1945.

If one sets $g_0 = g_1 = g$ and $C_0 = C_1 = C$, *i.e.*, maximum nonlinearity for both the barrier capacitance and conductance, the center of the circle is at $-0.5\ g$, $-0.5\ \omega C$ and the radius is $0.5\sqrt{g^2+\omega^2C^2}$. Thus, in this special case there is no low-frequency cutoff for negative IF conductance.

When g_β is not negative, W is of interest and is given by

$$W = \frac{(g_0 + g_a)^2 + (b_a + \omega C_0)^2}{g_a g_1^2}. \tag{47}$$

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The author wishes to thank A. Brodzinsky for his encouragement and helpful discussions.

Resonance Properties of Ring Circuits*

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Summary—**The ring guide or ring circuit, a microwave device consisting of a waveguide having the ends connected to form an annular ring, has properties similar to those of ordinary resonant cavities. Wave propagation within the ring guide, its interaction with a waveguide to which it is coupled, and its resonant circuit properties are investigated in this report. The properties of a prototype circuit consisting of a ring guide of rectangular cross section were found to agree with theory.**

Introduction

CAVITIES with conducting walls have, at wavelengths of the order of their geometrical sizes, the same electromagnetic properties as resonant circuits consisting of capacitances and inductances at lower frequencies. Therefore, cavities are useful as resonant circuits and filter elements at microwave frequencies. They can also be considered as waveguide sections short circuited at each end. The electromagnetic energy oscillates between the electric and magnetic states. Standing waves and an imaginary Poynting vector are of significance at any point in the cavity.

A new type of microwave circuit[1] consists of a waveguide having the ends connected to form a ring in which waves progressing in one direction only are excited. This circuit is characterized by a real Poynting vector within the ring guide cavity. Properties of the circuit, including the form of wave propagation within the ring, interaction with a waveguide to which it is coupled, and its Q-value, are investigated. Circuit performance when excited to produce traveling waves in one direction is compared with that obtained when excited to produce waves in both directions.

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[1] F. J. Tischer, Swedish Patent No. 152,491; August 26, 1952.

Wave Propagation in the Ring Circuit When Coupled to a Waveguide

The system under investigation consists of a waveguide to which a ring guide is coupled as shown schematically in Fig. 1. In order to obtain waves progressing in one direction only in the ring, directional coupling is used. When nondirectional coupling is applied, waves progressing in both directions are obtained. These two types of coupling are shown in Fig. 2, *i.e.*, directional coupling by two holes spaced a distance of $\lambda_g/4$ apart and nondirectional coupling by a single hole.

In the following derivation based on the wave concept, h indicates the waves progressing toward the coupling element, while r corresponds to waves reflected and traveling from the coupling element. The symmetry plane AA', Fig. 2, is used as phase reference. The ports of the main waveguide are 1 and 2, while those of the secondary guide which will be connected to form a ring are 3 and 4. The other parameters which describe the wave propagation in the region of the junction are the reflection coefficients ρ_{nn} at the four ports and the transmission coefficients T_{nm} between the different ports with reference to plane AA'.

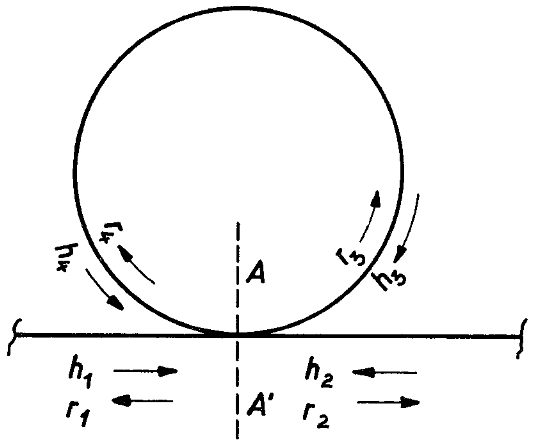

Fig. 1—Ring circuit.

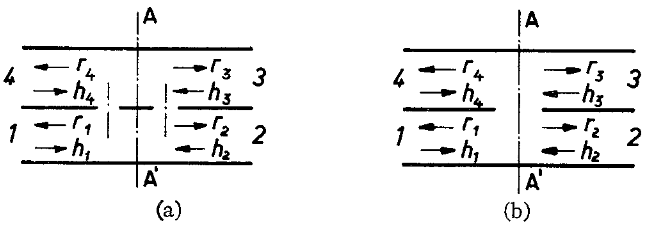

Fig. 2—Coupling conditions. (a) Directional coupling. (b) Non-directional coupling.

The wave propagation in the region of the junction can be described in general by the following equation systems:

$$\begin{aligned} r_1 &= \rho_{11}h_1 + T_{12}h_2 + T_{13}h_3 + T_{14}h_4 \\ r_2 &= T_{21}h_1 + \rho_{22}h_2 + T_{23}h_3 + T_{24}h_4 \\ r_3 &= T_{31}h_1 + T_{32}h_2 + \rho_{33}h_3 + T_{34}h_4 \\ r_4 &= T_{41}h_1 + T_{42}h_2 + T_{43}h_3 + \rho_{44}h_4. \end{aligned} \tag{1}$$

The assumption of symmetry of the coupling elements yields

$$\rho_{11} = \rho_{22} = \rho_{33} = \rho_{44} = \rho_0,$$
$$T_{nm} = T_{mn}$$

and

$$T_{12} = T_{34} = T, \quad T_{13} = T_{24} = k_h, \quad T_{14} = T_{23} = k_r$$

where k_h and k_r are coupling factors for the waves traveling in the two directions from the coupling element in the secondary guide. One obtains with these values

$$\begin{aligned} r_1 &= \rho_0 h_1 + k_h h_3 + k_r h_4 \\ r_2 &= T h_1 + k_r h_3 + k_h h_4 \\ r_3 &= k_h h_1 + \rho_0 h_3 + T h_4 \\ r_4 &= k_r h_1 + T h_3 + \rho_0 h_4 \end{aligned} \tag{2}$$

where h_2 is made 0 by reflectionless termination of port 2.

The ring guide is formed by connecting ports 3 and 4 with a length of guide so that:

$$\begin{aligned} h_3 &= r_4 \cdot e^{-\phi} \\ h_4 &= r_3 \cdot e^{-\phi}. \end{aligned} \tag{3}$$

ϕ is a complex quantity, the imaginary part being the phase delay around the ring guide and the real part the attenuation along this path in nepers. Thus,

$$\phi = (\alpha + i\beta)L \tag{4}$$

where L is the circumference and α and β are the attenuation and phase constants of the ring guide, respectively. Combining (3) and (2) yields:

$$\begin{aligned} r_3(1 - Te^{-\phi}) &= k_h h_1 + \rho_0 r_4 e^{-\phi} \\ r_4(1 - Te^{-\phi}) &= k_r h_1 + \rho_0 r_3 e^{-\phi}. \end{aligned} \tag{5}$$

Solving for r_3 and r_4,

$$r_3 = h_1 \frac{k_h(1 - Te^{-\phi}) + k_r\rho_0 e^{-\phi}}{(1 - Te^{-\phi})^2 - \rho_0^2 e^{-2\phi}} \tag{6}$$

and

$$r_4 = h_1 \frac{k_r(1 - Te^{-\phi}) + k_h\rho_0 e^{-\phi}}{(1 - Te^{-\phi})^2 - \rho_0^2 e^{-2\phi}} \tag{7}$$

are obtained. Similarly, r_1 and r_2 may be found. From these values, the reflection coefficient ρ_{in} and the transmission coefficient T_{12} in the main guide are

$$\rho_{\text{in}} = \frac{r_1}{h_1} = \rho_0 + \frac{2k_r k_h(1 - Te^{-\phi})e^{-\phi} + (k_r^2 + k_h^2)\rho_0 e^{-2\phi}}{(1 - Te^{-\phi})^2 - \rho_0^2 e^{-2\phi}} \tag{8}$$

and

$$T_{12} = \frac{r_2}{h_1} = T + \frac{2k_r k_h \rho_0 e^{-2\phi} + (k_r^2 + k_h^2)(1 - Te^{-\phi})e^{-\phi}}{(1 - Te^{-\phi})^2 - \rho_0^2 e^{-2\phi}}. \tag{9}$$

These relations describe the properties of the ring circuit as a resonant filter.

It is necessary to distinguish between two cases of wave propagation, namely, waves progressing in only one direction, and standing waves. These conditions depend on the coupling. They can be expressed by $k_r = 0$ for directional coupling and $k_h = k_r = k$ for nondirectional coupling.

For directional coupling,

$$R_D = \frac{r_3}{h_1 k_h} = \frac{1}{1 - Te^{-\phi}} \tag{10}$$

describes the complex wave in the ring guide. This relation is obtained for $k_r = 0$ and $\rho_0 = 0$ which follow from the properties of a directional coupling element. Therefore,

$$r_4 = 0, \qquad \rho_{\text{in}} = 0 \tag{11}$$

and

$$T_{12} = T + \frac{k_h^2 e^{-\phi}}{1 - Te^{-\phi}}. \tag{12}$$

Consideration of the nondirectional case ($k_h = k_r = k$) gives

$$R_N = \frac{r_3}{h_1 k} = \frac{r_4}{h_1 k} = \frac{1}{1 - (\rho_0 + T)e^{-\phi}} \tag{13}$$

and

$$\rho_{in} = \rho_0 + 2k^2 e^{-\phi} \frac{1}{1 - (\rho_0 + T)e^{-\phi}} \tag{14}$$

$$T_{12} = T + 2k^2 e^{-\phi} \frac{1}{1 - (\rho_0 + T)e^{-\phi}} \cdot \tag{15}$$

From (10) through (15), it can be seen that the relations for the amplitude of the ring wave represented by R_D and R_N are the most important. They appear in all relations for the over-all reflection and transmission coefficients. This is expected since a part of the energy in the ring is coupled back into the main guide. This energy adds to the energy in the main guide, and the resultant interaction determines the properties of the circuit.

Frequency Dependence of the Wave Amplitude in the Ring Guide

The resonance characteristics can be obtained by representing the wave amplitude in the ring guide as a function of frequency. Accordingly, the absolute values $|R_D|$ and $|R_N|$ must be determined. Introduction of the attenuation and phase constants, α and $\beta = 2\pi/\lambda_g$ into (10) and (13) gives

$$R_D = \frac{1}{1 - Te^{-\alpha L}e^{-i\frac{2\pi_L}{\lambda_g}}} \tag{16}$$

and

$$R_N = \frac{1}{1 - (\rho_0 + T)e^{-\alpha L}e^{-i\frac{2\pi_L}{\lambda_g}}} \cdot \tag{17}$$

The equations show that near resonance, variation of the frequency results mainly in a rotation of the complex terms T and $\rho_0 + T$ in the denominators of both relations. The frequency dependence follows the same law for both coupling conditions. This dependence can be represented in a normalized form valid for both coupling conditions by

$$R = \frac{1}{1 - \alpha e^{-ix}} \cdot \tag{18}$$

Plotting the absolute value $|R|$ vs frequency deviation gives a resonance curve as I in Fig. 3. Resonance occurs when x equals a multiple of 2π, the circumference L then being a multiple of the guide wavelength λ_g.

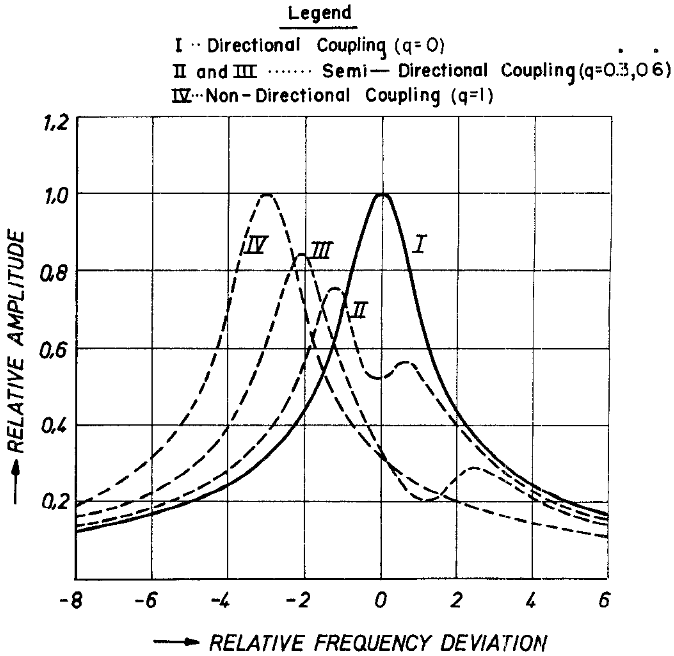

Fig. 3—Frequency response.

Calculation of the reflection coefficient ρ_0 in the nondirectional case for small and lossless coupling shows that it is small in magnitude and approximately a pure imaginary. R_D and R_N were found to differ mainly in their resonant frequencies.

The Case of Semidirectional Coupling

Interesting relations are obtained if the coupling in the two directions, defined by the coupling factors k_h and k_r, is unequal. This type of coupling is called semidirectional. If $k_r/k_h = q$ is introduced in (6),

$$r_3 = h_1 k_h \frac{1 - \left(T - \frac{k_r}{k_h}\rho_0\right)e^{-\phi}}{[1 - (T + \rho_0)e^{-\phi}][1 - (T - \rho_0)e^{-\phi}]} \tag{19}$$

is obtained. Eq. (19) has two poles and a zero between them in the complex plane. Therefore, the frequency response of a ring circuit with semidirectional coupling has a form which is analogous to that of band-pass filters containing two resonant circuits.

Eq. (19) is plotted in Fig. 3 for $q = 0$, $0.3\dot{3}$, $0.6\dot{6}$, 1, for curves I through IV, respectively, under the condition $|\rho_{0_{max}}| = 3(1 - |T|)$. Ordinary resonance curves for directional and nondirectional coupling and the double peak response for semidirectional coupling are shown.

A similar double peak response is observed when a discontinuity is introduced in a directionally coupled ring circuit with waves traveling in only one direction. To analyze this case, the assumption is made wherein a symmetrical discontinuity with a reflection coefficient ρ_1 is placed opposite to the coupling element (Fig. 4). The discontinuity can be taken into account by introducing

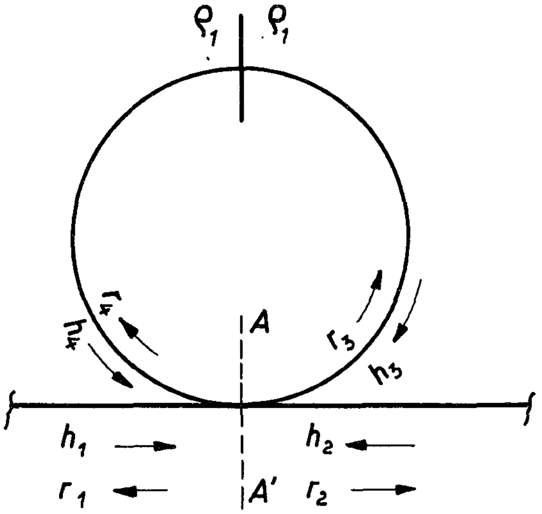

Fig. 4—Ring circuit with discontinuity (ρ_1).

$$h_3 = [r_3\rho_1 + r_4(1 + \rho_1)]e^{-\phi}$$
$$h_4 = [r_4\rho_1 + r_3(1 + \rho_1)e^{-\phi}] \quad (20)$$
$$r_3 = h_1 \frac{k_h[1 - \rho_0\rho_1 e^{-\phi} - T(1 + \rho_1)e^{-\phi}] + k_r[\rho_0(1 + \rho_1)e^{-\phi} + T\rho_1 e^{-\phi}]}{[1 - \rho_0\rho_1 e^{-\phi} - T(1 + \rho_1)e^{-\phi}]^2 - [\rho_0(1 + \rho_1)e^{-\phi} + T\rho_1 e^{-\phi}]^2}. \quad (21)$$

The corresponding relation for r_4 can be obtained by interchanging k_h and k_r in (21). For the case of directional coupling where $k_r = 0$ and $\rho_0 = 0$, (21) can be simplified to:

$$r_3 = h_1 k_h \frac{1 - T(1 + \rho_1)e^{-\phi}}{[1 - Te^{-\phi}][1 - T(1 + 2\rho_1)e^{-\phi}]}. \quad (22)$$

Comparison of (22) and (19) shows that both relations have characteristics similar to those of a band-pass filter with two resonant circuits. A difference exists, since (22) gives a symmetrical response having equal frequency deviations from the zero to the poles while (19) is nonsymmetrical.

The Q-Value of Resonant Ring Circuits

When the ring circuit is used as a resonator, the quality factor can be obtained by finding the frequency deviations $\Delta\lambda_{1,2}$ from resonance for which the amplitude of the ring waves decreases by a factor $1/\sqrt{2}$. Starting from the normalized frequency response, according to (18), the reciprocal value of R is

$$\frac{1}{R} = 1 - ae^{-ix}. \quad (23)$$

For resonance, $x = 2\pi, 4\pi, \cdots n2\pi$,

$$\frac{1}{R_{res}} = 1 - a \quad (24)$$

is obtained where $a = |T|e^{-\alpha L}$ or $|T + \rho_0|e^{-\alpha L}$. For small attenuation in the ring guide and loose coupling $(1 - a) \ll 1$, (23) can be approximated by the first two terms of a Taylor's series for frequencies near resonance. This yields

$$\frac{1}{R} \approx 1 - a + iae^{-ix}\frac{dx}{d\lambda}\Delta\lambda. \quad (25)$$

Use of this simplified relation allows determination of $\Delta\lambda_{1,2}$ by equating real and imaginary parts of (25). Therefore,

$$\pm\Delta\lambda_{1,2} \approx \frac{1 - a}{a} \frac{1}{\frac{dx}{d\lambda}} \quad (26)$$

where $e^{-ix} = 1$ at resonance.

Introduction of $x = 2\pi L/\lambda_g$ and differentiation gives

$$\pm\frac{\Delta\lambda_{1,2}}{\lambda_{res}} \approx \frac{1 - a}{a}\left(\frac{\lambda_{res}}{\lambda_{g\,res}}\right)^2 \frac{1}{\beta L} \quad (27)$$

which is valid for both coupling conditions of the ring circuits at resonance. If a is replaced by the values corresponding to the coupling conditions, the following results are obtained:

Directional coupling: $a = |T|e^{-\alpha L} \approx |T|(1 - \alpha L)$

$$a = |T|e^{-\alpha L} \approx |T|(1 - \alpha L)$$

$$\pm\frac{\Delta\lambda_{1,2}}{\lambda_{res}} \approx \left(\frac{1 + \alpha L}{|T|} - 1\right)\left(\frac{\lambda_{res}}{\lambda_{g\,res}}\right)^2 \frac{1}{\beta L}. \quad (28)$$

Nondirectional coupling: $a = |T + \rho_0|e^{-\alpha L}$

$$\pm\frac{\Delta\lambda_{1,2}}{\lambda_{res}} \approx \left(\frac{1 + \alpha L}{|T + \rho_0|} - 1\right)\left(\frac{\lambda_{res}}{\lambda_{g\,res}}\right)^2 \frac{1}{\beta L}, \quad (29)$$

where β is the resonant phase constant. The relative wavelength deviations (28) and (29) are measures of the Q values. The unloaded values are obtained by reducing the coupling zero ($\rho_0 \to 0$, $|T| \to 1$). Under this condition,

$$\pm\frac{\Delta\lambda_{1,2}}{\lambda_{res}} \approx \frac{\alpha}{\beta}\left(\frac{\lambda_{res}}{\lambda_{g\,res}}\right)^2 \quad (30)$$

is valid for both types of coupling, giving for the Q value

$$Q = \frac{\lambda_{res}}{2\Delta\lambda_{1,2}} \approx \frac{\beta}{2\alpha}\left(\frac{\lambda_{g\,res}}{\lambda_{res}}\right)^2. \quad (31)$$

This unloaded Q value can also be obtained directly by use of the energy concept applied to the definition of the Q value. According to this definition

$$Q = \frac{\omega W_{tot}}{P_d} \quad (32)$$

where $\omega = 2\pi f$, W_{tot} corresponds to the total energy content of the circuit, and P_d to the power dissipated. The power dissipated in a guide of length L is

$$P_d = P_{in} - P_{out} = P_{in}(1 - e^{-2\alpha L}) \approx 2\alpha L P_{in} \quad (33)$$

for small αL, where P_{in} and P_{out} are the powers passing through the input and output cross sections. The power transported by the waves through the input cross section can be expressed as the product of the energy contained per unit length in the guide w_{tot} and the group velocity v_{gr}. Thus

$$P_{in} = w_{tot} \cdot v_{gr} = w_{tot} \frac{v_0^2}{v_{ph}} \quad (34)$$

introducing

$$\lambda_0 = \frac{v_0}{f}, \qquad \lambda_g = \frac{v_{ph}}{f}$$

gives

$$W_{tot} = w_{tot} \cdot L = \frac{P_{in} \cdot L \cdot v_{ph}}{v_0^2}$$

from which

$$Q = \frac{1}{2\alpha} \frac{2\pi}{\lambda_g} \left(\frac{\lambda_g}{\lambda_0}\right)^2. \quad (35)$$

By introduction of $\beta = 2\pi/\lambda_g$, (31) may be obtained. In both derivations, the assumption was made that the attenuation varies insignificantly in the region of resonance.

Experimental Investigation of a Ring Circuit with Rectangular Waveguide

The ring guide permits design of a circuit with variable resonant frequency without use of sliding shorts and capacity plungers. One type of ring circuit comprising a rectangular waveguide of variable width is shown in Fig. 5. The waveguide in the form of a ring consists of two grooves of rectangular cross section machined into adjacent walls of two thick metallic disks. Each groove forms half of the guide in which TE_{01} waves with radial E vector are excited. One of the disks is movable in the axial direction. The movement is obtained by rotation of a micrometer head to which one disk is attached, thus permitting a change of the width of the guide. The change of width results in a variation of the wavelength in the guide λ_g, and of the resonant wavelength. The resonant wavelength can be expressed by

$$\lambda_{res} \approx \frac{1}{\sqrt{\frac{n^2}{L^2} + \frac{1}{4a^2}}}$$

where a is the width, L the circumference of the guide, and n the number of wavelengths per circumference. Normalization with respect to L yields

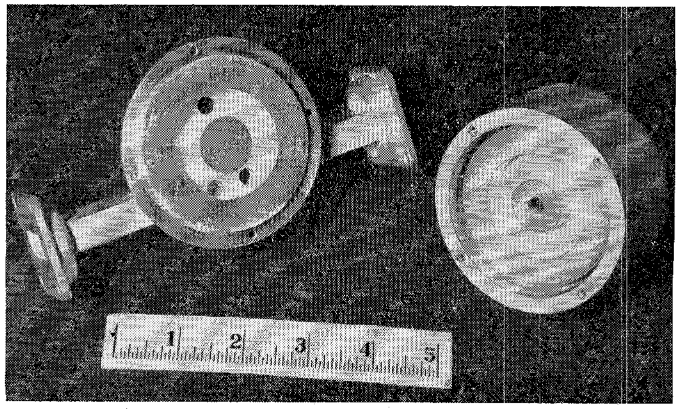

Fig. 5—Prototype of a ring circuit with rectangular waveguide and variable resonance frequency.

$$\frac{\lambda_{res}}{L} \approx \frac{1}{\sqrt{n^2 + \left(\frac{L}{2a}\right)^2}} \quad (36)$$

which relates resonant wavelength to the dimensions of the ring guide. Eq. (33) is an approximation in which the bending of the guides and the influence of the slots are neglected.

The Q value of a ring guide is given by (31). The values of α and β for a rectangular waveguide are

$$\alpha = \frac{(6.9)10^{-6}\sqrt{f^{[c/s]}}}{\lambda_0^{[m]}} \frac{\frac{\lambda_0}{b} + 4\left(\frac{\lambda_0}{\lambda_c}\right)^3}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} \sqrt{\frac{\sigma}{\sigma_{cu}}} \quad (37)$$

and

$$\beta = \frac{2\pi}{\lambda_g},$$

where a is the variable width, b the constant height of the guide, and σ/σ_{cu} the conductivity related to that of copper. Introduction in (31) yields

$$\frac{2\Delta f}{f_{res}} = \frac{2\Delta f}{\lambda_{res}} = (2.2) \cdot 10^{-10} m \sqrt{f} \sqrt{\frac{\sigma_{cu}}{\sigma}},$$

$$m = \frac{\lambda_0}{b} + 4\left(\frac{\lambda_0}{\lambda_c}\right)^3, \quad (38)$$

the form factor m showing the dependence of Q on the geometry of the ring guide. In Fig. 6, m is shown as a function of the normalized wavelength in the guide λ_0/λ_g with λ_0/b as the parameter. The diagram permits the choice of dimensions to obtain a high Q value which is inversely proportional to Δf. It should be noted that the half-power bandwidth of a comparable circular cavity in the case of optimized dimensions corresponds to a form factor $m = 1.5$ for low loss TE_{01} waves. The measured Q values of the ring circuit shown in Fig. 5 are compared with those obtained using (38) in Fig. 7. Taking into account the fact that the measured values

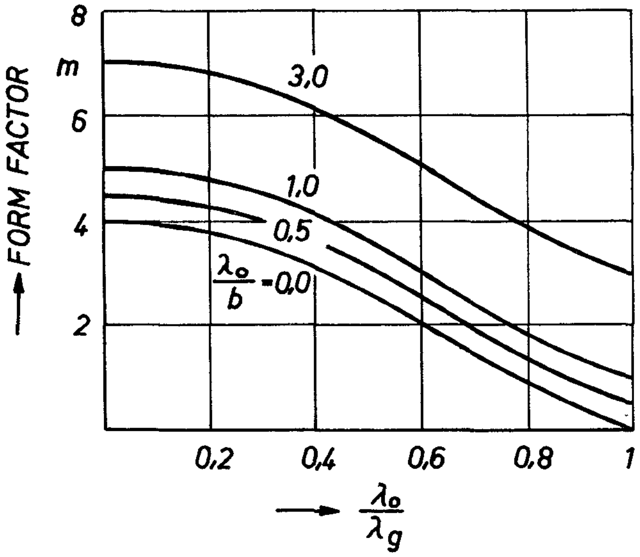

Fig. 6—Dependence of the form factor m on the geometry of rectangular waveguide.

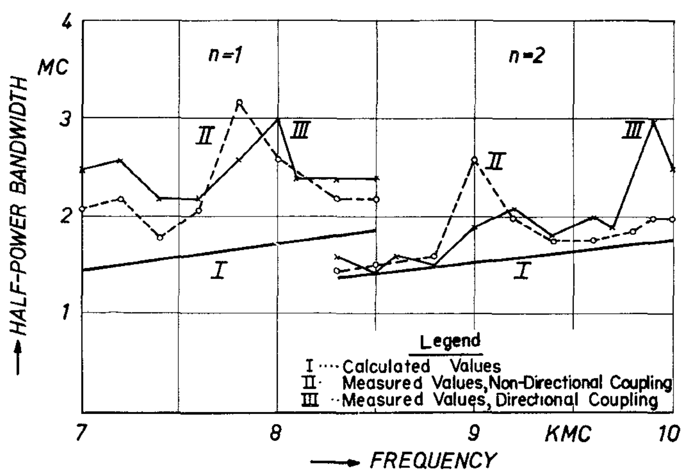

Fig. 7—Calculated and measured values of the half-power bandwidth for prototype shown in Fig. 5.

of Δf correspond to the loaded Q while the calculation is based on the unloaded Q, the agreement is considered satisfactory.

Conclusion

The essential difference between the ring circuit and the conventional microwave cavity lies in its capacity to resonate with unidirectional progressive waves in the ring when directionally coupled to the main guide. When nondirectionally coupled, the circuit assumes the behavior of an ordinary cavity with standing waves. The resonance curves are similar in either case, the Q values differing by only a small amount due to the coupling effects.

Interesting properties result from semidirectional coupling or the introduction of a small discontinuity into the directionally coupled ring. In either instance, the circuit is characterized by a double-peaked resonance. Accordingly, it is useful as a microwave filter.

The device has immediate application as a tunable resonant circuit. Tuning is accomplished by any means whereby the electrical length of the ring is changed. Thus a phase shifter may be employed, permitting a variation in the phase of the guide wavelength around the ring so that its electrical length can be made a multiple of a wavelength at resonance. An adjustable width ring, which allows adjusting the guide wavelength uniformly over the circumference of the ring, is a means of performing simple, low-loss tuning. Since increasing the ring width reduces the attenuation, higher modes of resonance will have higher Q values. Multiple resonance occurs when the guide is adjusted for two or more guide wavelengths.

Possible fields of application for the ring circuit are the same as those of commonly used standing-wave type cavities. The high Q values of the adjustable width circuit favor its application for frequency measurement and stabilization. The geometrical form permits easy fabrication, especially for use at extremely high frequencies.

Acknowledgment

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